VALIDITY OF THE QUASIHOMOGENEOUS APPROXIMATION FOR AN ANALYSIS OF RADIATIVE HEATING IN AN AEROSOL STREAM

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The thermal resistances in a stream are analyzed and the validity limits of the quasihomogeneous approximation are established, on the basis of information about the temperature of the phases during radiative heating.

In view of the trend toward higher operating temperatures in many heat and mass transfer processes, with the advantages of using the radiative mechanism for heating a gaseous heat carrier, studies have been made recently concerning the interaction between a high-temperature radiator and a gas stream which carries finely-dispersed solid particles [1, 2, 3]. The suitability of such gaseous suspensions derives, essentially, from the fact that most mono- and diatomic gases are diathermal within the practically important temperature range up to 5000°K and that, therefore, adding a definite amount of finely-dispersed solid particles of a highly absorptive material ensures the necessary absorption characteristics of the stream.

We may also point out a similar process occurring in nature, namely the interaction between solar radiant heat and the dusty atmospheres of Earth, Mars, etc. For calculating the heat transfer from the radiator to the gas in such systems, one often uses the model of a quasihomogeneous medium with effective thermophysical and optical properties. This convenient and in many cases sufficiently accurate approach has, however, also the following shortcomings.

The complex process of heat transfer is simplified here by disregarding the effect of radiation scatter at the particles, and yet the amplitude of the real scatter function is sometimes quite large [3]. It is not possible to account for the true concentration field of a stream and for the polydispersivity of the dispersing phase. Any effect of the interphase heat transfer on the rate of change of particle enthalpy is ignored completely, along with the rate of net heat transfer from the radiator to the gas. The significance of the latter factor will be analyzed in this article.

Establishing the validity limits of the quasihomogeneous approximation requires a comparative analysis of the process rates at various stages of heat transfer from the radiator to the gas. The homogeneous model is obviously unsuitable for this purpose, in principle, and it is necessary to consider the actual system structure where solid aerosol particles serve essentially as the intermediate heat carrier. In this case radiative heating of the gas is effected in several steps: the thermal radiation is absorbed by particles during the first stage, part of the heat then raises their enthalpy and part is transmitted to the gas by the interphase heat transfer mechanism. The first stage will not be considered here any further, as it has been treated already in many studies dealing with the problem under substantial simplifications [4, 5]. Both the second and the third stage have been explored to a much lesser extent, and the approximation of existing solutions to the problem is thus compounded [6, 7]. In view of this, an approximate theoretical and numerical analysis was performed in [8] on a more complete basis, and the results will be used here.

This analysis is based on the knowledge of processes occurring in a typical heterogeneous cell, such a cell model having been proposed earlier for the study of gaseous dispersions flowing through ducts [9].

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Under the conditions of our problem, the cell boundaries are adiabatic and, under certain stipulations (that the particles be sufficiently small to limit the interphase heat transfer to conductive heat transfer only, that the particles be spherical and equal in size, that the properties of both phases vary very little during heating, and that the radiation emitted by particles be negligible in comparison with the radiation absorbed), the conduction of heat is described by the following system of equations:

$$\begin{gathered}\frac{\partial T}{\partial \tau}=a_{\mathrm{s}}\left(\frac{\partial^2 T}{\partial r^2}+\frac{2}{r}\frac{\partial T}{\partial r}\right),\ 0\leqslant r\leqslant r_{\mathrm{s}};\\ \frac{\partial T}{\partial \tau}=a\left(\frac{\partial^2 T}{\partial r^2}+\frac{2}{r}\frac{\partial T}{\partial r}\right),\ r_{\mathrm{s}}\leqslant r\leqslant r_{\mathrm{c}};\\ \text{for }\ \tau=0\quad T=T_{\text{н}},\ r_{\mathrm{c}}=r_{\mathrm{s}}\sqrt[3]{1+\beta^{-1}};\\ \left.\frac{\partial T}{\partial r}\right|_{r=0}=\left.\frac{\partial T}{\partial r}\right|_{r=r_{\mathrm{c}}}=0;\ \lambda_{\mathrm{s}}\left.\frac{\partial T}{\partial r}\right|_{r=r_{\mathrm{s}-}}=\lambda\left.\frac{\partial T}{\partial r}\right|_{r=r_{\mathrm{s}+}}+\frac{1}{4}k_f q_R.\end{gathered}\tag{1}$$

This system, in the dimensionless version, was solved numerically on a model M-220 computer. As a result, we have obtained data on the trends of local and mean temperatures in both phases, on the Nusselt number as a function of the heating time, on the concentration of particles, and on the thermophysical properties of both phases. Assuming a zero-gradient temperature field inside a particle, we have simplified system (1) substantially and have obtained an approximate analytical solution by the Goodman integral method [8]. When the original heat wave has reached the cell boundaries and a temperature field has been established, then the mean temperatures of both phases can be described by the following relations:

$$\bar{\vartheta}_{\mathrm{s}}=\frac{3\mathrm{Fo}^*z}{1+z}+\frac{2}{\mathrm{Nu}\,(1+z)^2}\left\{1-\exp\left[-\frac{3}{2}\,\mathrm{NuFo}^*(1+z)\right]\right\};\tag{2}$$

$$\bar{\vartheta}_{\mathrm{s}}-\bar{\vartheta}=\frac{2}{\mathrm{Nu}\,(1+z)}\left\{1-\exp\left[-\frac{3}{2}\,\mathrm{NuFo}^*(1+z)\right]\right\}.\tag{3}$$

Here

$$\begin{gathered}\vartheta=\frac{4\lambda\,(T-T_{\text{н}})}{k_f q_R r_{\mathrm{s}}};\ \mathrm{Fo}^*=\frac{\lambda\tau}{r_{\mathrm{s}}^2 c_{\mathrm{s}}\rho_{\mathrm{s}}};\\ z=\frac{c_{\mathrm{s}}}{c}\,\mu;\ \mathrm{Nu}=2\,\frac{(r_{\mathrm{c}}+r_{\mathrm{s}})(r_{\mathrm{c}}^2+r_{\mathrm{c}}r_{\mathrm{s}}+r_{\mathrm{s}}^2)}{r_{\mathrm{s}}^3-0{,}25r_{\mathrm{s}}\,(r_{\mathrm{c}}+r_{\mathrm{s}})^2}=\mathrm{const}.\end{gathered}\tag{4}$$

According to the numerical solution, the Nusselt number decreases rapidly from infinity toward the threshold level $\mathrm{Nu}_\infty \simeq 2 + 3\beta^{0.28}$ as the Fourier number increases, which agrees also with Eq. (4), and it thus exceeds the usually accepted value 2.0 by an amount which increases with the concentration of particles in the stream. The mean phase temperatures calculated numerically agree with those based on formulas (2) and (3) beginning already at $\mathrm{Fo}^* \simeq 10^{-2}$. Moreover, the initial process stage is characterized by an almost constant gas temperature leading the temperature of heated particles. This information about a transient (in terms of conductive heat transfer) of a cell allows us to evaluate the significance of the various process stages as well as the total and the component thermal resistances. We will apply the conventional method of comparative analysis to each component thermal resistance, for an evaluation of the net heat transfer and for pinpointing that process which limits its rate. From the viewpoint of the homogeneous model, only radiative heat transfer imposes some limitations on the heating rate. Let us consider this from the viewpoint of the heterogeneous model. If Q denotes the amount of heat transmitted per unit time from the radiator to the gas and F^* denotes some conveniently defined effective surface area in the apparatus, then the rate of the resulting process can be evaluated in terms of a quantity whose significance is that of a transmission coefficient k. The thermal resistance R_T will then be inversely proportional to this coefficient k:

$$k\equiv\frac{Q}{F^*(T_0-T)};\ R_T\equiv\frac{1}{kF^*}=\frac{T_0-T}{Q}.\tag{5}$$

This amount of heat Q is transmitted from the particles to the gas by the mechanism of interphase heat transfer,

$$Q=\alpha F_{\mathrm{s}}\,(T_{\mathrm{s}}-T);\ R\equiv\frac{1}{\alpha F_{\mathrm{s}}}=\frac{T_{\mathrm{s}}-T}{Q}.\tag{6}$$

The amount of heat radiated from the source and absorbed by the particles is

$$Q_{\mathrm{r}} = Q + Q_{\mathrm{s}} = \alpha_{\mathrm{r}} F_{\mathrm{s}} (T_0 - T_{\mathrm{s}});\ R_{\mathrm{r}} \equiv \frac{1}{\alpha_{\mathrm{r}} F_{\mathrm{s}}} = \frac{T_0 - T_{\mathrm{s}}}{Q_{\mathrm{r}}}. \tag{7}$$

Here Q_S is determined from the change of enthalpy of solid particles, in accordance with the heat balance

$$\frac{Q_{\mathrm{s}}}{Q_{\mathrm{r}}} = z\frac{\delta T_{\mathrm{s}}}{\delta T};\ \frac{Q_{\mathrm{r}}}{Q} = 1 + z\frac{\delta T_{\mathrm{s}}}{\delta T} = \frac{k_f q_R}{4\alpha (T_{\mathrm{s}} - T)} = \frac{2}{\mathrm{Nu}(\vartheta_{\mathrm{s}} - \vartheta)}. \tag{8}$$

Let us express the total thermal resistance R_T in terms of its components. For this purpose we write (5) as

$$R_{\mathrm{T}} = \frac{T_0 - T}{Q} = \frac{T_0 - T}{Q_{\mathrm{r}}}\left(1 + \frac{Q_{\mathrm{s}}}{Q}\right) + \frac{T_{\mathrm{s}} - T}{Q}. \tag{9}$$

According to the definition of the component thermal resistances (6)-(7), we have

$$R_{\mathrm{T}} = R_{\mathrm{r}} + R_{\mathrm{s}} + R, \tag{10}$$

$$R_{\mathrm{s}} = R_{\mathrm{r}}\frac{Q_{\mathrm{s}}}{Q} = R_{\mathrm{r}} z \frac{\delta T_{\mathrm{s}}}{\delta T}. \tag{11}$$

The term R_S accounts for the decrease in thermal flux due to heat accumulation in the particles and it is equivalent to the thermal resistance of the particles, referred to conditions of sequential heat propagation. Since in the steady-state heating stage ($\tau \to \infty$) $\delta T_S = \delta T$, hence

$$R_{\mathrm{s}\infty} = R_{\mathrm{r}} z;\ R_{\mathrm{T}\infty} = R_{\mathrm{r}}(1 + z) + R. \tag{12}$$

On the other hand, in the initial stage there appears an additional thermal resistance due to prior heat accumulation in the particles

$$\Delta R_{\mathrm{s}} = R_{\mathrm{s}} - R_{\mathrm{s}\infty} = R_{\mathrm{r}} z \left(\frac{\delta T_{\mathrm{s}}}{\delta T} - 1\right). \tag{13}$$

For the individual components of the total thermal resistance, referred to the resistance to radiative heat transfer, we have

$$\frac{R}{R_{\mathrm{r}}} = \frac{\alpha_{\mathrm{r}}}{\alpha} = \frac{q_R k_f}{4(T_0 - T_{\mathrm{s}})\alpha} = \frac{2}{\mathrm{Nu}(\vartheta_0 - \vartheta_{\mathrm{s}})};\ \frac{R_{\mathrm{s}\infty}}{R_{\mathrm{r}}} = z. \tag{14}$$

From (8) and (13) follows

$$\frac{\Delta R_{\mathrm{s}}}{R_{\mathrm{r}}} = \frac{2}{\mathrm{Nu}(\vartheta_{\mathrm{s}} - \vartheta)} - (1 + z). \tag{15}$$

Then

$$\frac{R_{\mathrm{T}}}{R_{\mathrm{r}}} = \frac{2}{\mathrm{Nu}}\left(\frac{1}{\vartheta_{\mathrm{s}} - \vartheta} + \frac{1}{\vartheta_0 - \vartheta_{\mathrm{s}}}\right). \tag{16}$$

The quasihomogeneous approximation corresponds to the extreme case of an infinite rate of interphase heat transfer ($\mathrm{Nu} \to \infty$) and equal heating rates of both phases $\delta T_S = \delta T$. Then, according to expressions (14)-(16),

$$\frac{R^*}{R_{\mathrm{r}}} = 0;\ \frac{R_{\mathrm{s}}^*}{R_{\mathrm{r}}} = z;\ \frac{R_{\mathrm{T}}^*}{R_{\text{л}}} = 1 + z. \tag{17}$$

Unlike in a quasicontinuous medium, therefore, in a real gaseous suspension there appear two additional resistances to heat transfer from radiator to gas: they are a consequence of the finite rate of thermal interaction between both phases, namely a resistance R/R_r to direct interphase heat transfer and a resistance $\Delta R_S/R_r$ due to prior heating of the particles in the initial process stage.

In order to illustrate the significance of these two additional thermal resistances, we have shown some results of calculations in Figs. 1 and 2. The components of the total thermal resistance R_i/R_r are shown in Fig. 1a for a system of graphite particles suspended in nitrogen (N_2–C) under a pressure P = 1 bar, at a temperature T = 1000°K, and with a relative mass flow rate μ = 1 kg/sec/kg/sec, as a function of the Fourier number (dimensionless process time) and determined on the basis of the approximate theoretical solution (top curve) as well as on the basis of numerical calculations on a digital computer (bottom

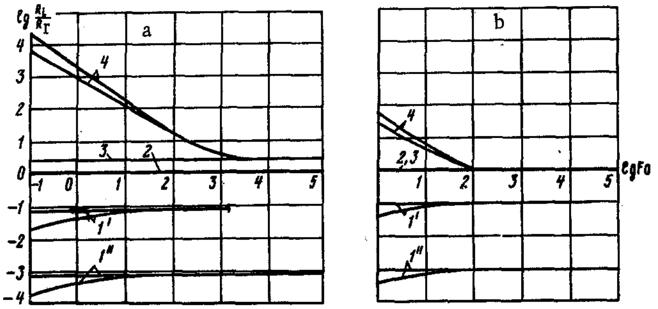

Fig. 1. Relative thermal resistances as functions of the Fourier number Fo: 1') R/R_r ($\vartheta_0 = 10$); 1") R/R_r ($\vartheta_0 = 10^3$); 2) $R_r/R_r \equiv (R_{s\infty} + R_r)/R_r = 1 + z$; 4) $(R_r + R_{s\infty} + \Delta R_{s\infty})/R_r$.

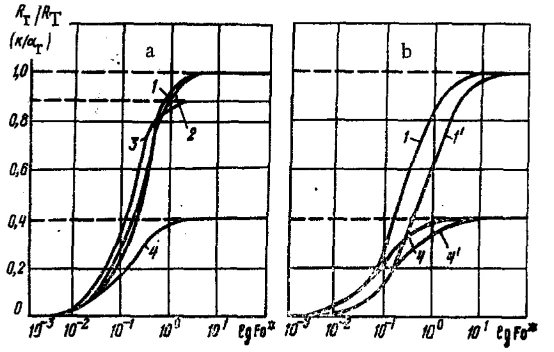

Fig. 2. Total thermal resistance as a function of the Fourier number Fo*.

curves). The trend of the curves calculated for other conditions in the N_2–C system was similar. It must be noted that the results obtained by both methods are in satisfactory agreement, confirming the validity of formulas (2) and (3) for $Fo^* \geq 10^{-1}$.

The dimensionless temperature of the radiator, evaluated under various conditions, is $\vartheta_0 \gg 10$. The relative resistance R/R_r is here always low and much less than unity (curves 1). The relative resistance to radiative heat transfer is equal to unity, by definition, and independent of the Fourier number Fo (curve 2), and is the limiting thermal resistance of particles $R_{s\infty}/R_r$ equal to z (curve 3). Moreover, the additional resistance $\Delta R_s/R_r$ is quite appreciable in the low range of the Fourier number ($\Delta R_s/R_r > 1$, curve 4) and vanishes as the heating mode becomes regular with time. Analogous curves plotted for a system of submicron tungsten particles in hydrogen (P = 100 bars, T = 2000°K, $\mu = 0.01$ kg /sec/kg/sec) are shown in Fig. 1b.

In Fig. 2a, b are shown, respectively, the local values of $R_r/R_T = kF^*/\alpha_r F_s$ and those averaged over the process time, i.e., of the heat transfer rate referred to the rate of radiative heating α_r of particles. On the same diagram is also shown the R_r/R_T^* curve based on the quasihomogeneous approximation according to formula (17).

The relative thermal resistance was averaged as follows:

$$\frac{\overline{k}}{\alpha_r} = \frac{\mathrm{Nu}}{2Fo^*} \int\limits_0^{Fo^*} \frac{dFo^*}{(\vartheta_0 - \vartheta_s)^{-1} + (\vartheta_s - \vartheta)^{-1}} = \frac{1}{1+z} - \frac{2}{3(1+z)^2 \mathrm{Nu}} \left\{ 1 - \exp\left[-\frac{3}{2} \mathrm{Nu} Fo^* (1+z) \right] \right\}.$$

When $F^* = F_s$, according to the data, then R_r/R_T equal to k/α_r varies throughout the heating process, until at a certain $Fo^* = Fo^*_{lim}$ a constant final level is reached which corresponds to a transition to regular radiative heating of the gas by the particles. The numerical value of this final resistance ratio, always less than unity, depends on the characteristics of the aerosol system $(R_r/R_T)_\infty = (1 + z)^{-1}$. For instance, for hydrogen–tungsten (H_2–W) systems it is very close to unity (curves 1 and 2 in Fig. 2a) so that $z = 10^{-4}$ or 10^{-3} ($\mu = 10^{-2}$ or 10^{-1}, respectively), while for nitrogen–graphite (N_2–C) systems $(k/\alpha_r)_\infty = 0.875$ or 0.4 when $\mu = 10^{-1}$ (curve 3) or 1.0 kg/sec/kg/sec (curve 4), respectively, with z = 0.15. The average values of k/α_r are the same, but after some longer period of time they increase less with increasing Fo* than the instantaneous values (curves 1' and 4' in Fig. 2b), which corresponds to the usual relation between average and instantaneous values.

A comparison between the dashed and the solid curves indicates that in each case (Fig. 2a, b) the quasihomogeneous approximation (dashed lines) yields for the initial process stage (small values of Fo) a value for the rate of gas heating which becomes increasingly too high as Fo* drops further below Fo^*_{lim}. Characteristically, Fo^*_{lim} does not depend much on the properties of the dispersion system and changes slightly, as has been noted earlier, when a conversion is made from instantaneous to average values. Thus, under the conditions $P = 1$ or 100 bars, $T = 2000°K$, $\mu = 10^{-2}$ to 10, and $\vartheta_0 = 10$ to 10^3, one may assume that approximately $Fo^*_{lim} \simeq 1$ for the instantaneous values of $(k/\alpha_T)(R_T/R_T)$ and $Fo^*_{lim} \simeq 10$ for the average values $\bar{k}/\alpha_T$. Only at $Fo^* > Fo^*_{lim}$ does the quasihomogeneous approximation become valid, inasmuch as now the process is regular and $\Delta R_s/R_r = 0$ and the resistance to intrinsically interphase heat transfer becomes negligible.

Thus, when calculating relatively short processes of radiative heating in a dispersion system, replacing such a system by a homogeneous medium with effective properties can lead to errors which become greater at smaller values of Fo^* below $Fo^*_{lim} = 1$ (for instantaneous values of the relative rate of heat transfer from radiator to gas) or $Fo^*_{lim} = 10$ (for average values $\bar{k}/\alpha_r$). In those cases, calculations and analysis of the heating process must take into account the heterogeneous structure of the stream and the finite rate of interphase heat transfer. The quasihomogeneous approximation may be considered valid at $Fo^* \geq Fo^*_{lim}$.

These results are based on the earlier assumptions and, as has been mentioned already, do not apply to the case of radiative heat transfer from particles; the purpose of this analysis was to emphasize those stages in the process of gas heating by solid particles which are usually disregarded in conventional analysis. Obviously, an analysis of heat radiation in the heterogeneous approximation presents a separate and even more difficult task.

Another not less important problem has not been considered here, namely an analysis of this process with changes in the aggregate state of solid sols, i.e., their fusion and subsequent evaporation, also taken into account. It is evident, furthermore, that the results obtained here are valid only for systems with the Knudsen number $Kn < 10^{-2}$, which is the case under the process conditions which have been considered here.

NOTATION

T is the temperature;
τ is the time;
r_s is the particle radius;
r_c is the cell radius;
β is the true relative volume flow rate;
μ is the true relative mass flow rate;
λ is the thermal conductivity;
a is the thermal diffusivity;
Nu is the Nusselt number;
Fo is the Fourier number;
c is the specific heat;
α is the heat transfer coefficient;
k is the heat transmission coefficient;
F is the surface area;
R is the thermal resistance;
Q is the thermal flux;
k_f is the absorptivity.

Subscripts

s denotes the solid phase;
T denotes total;
i denotes initial temperature.

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